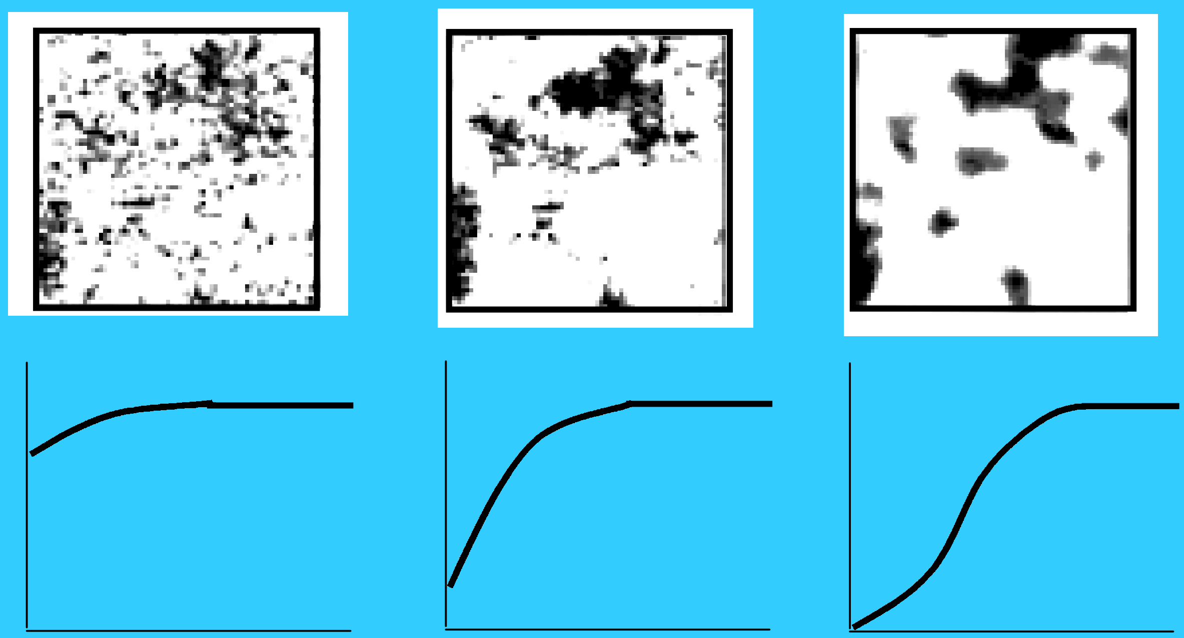
Variogram Calculation and Interpretation

- Spatial Statistics
- Coordinate and Data Transformation
- Define the Variogram
- How to Calculate Variograms
- “Visual Calibration”
- Variogram Interpretation
- Show Expected Behavior
- Work Through Some Examples
- Test Your Understanding

Spatial Statistics



- Spatial variability/continuity depends on the detailed distribution of the petrophysical attribute; our measure must be customized for each field and each attribute (ϕ,K)
- Depending on the level of diagenesis, the spatial variability may be similar within similar depositional environments. The recognition of this has led to outcrop studies.

Data Transformation

Why do we need to worry about data transformation?

- Attributes, such as permeability, with highly skewed data distributions present problems in variogram calculation; the extreme values have a significant impact on the variogram.
- One common transform is to take logarithms,

$$y = \log_{10}(z)$$

perform all statistical analyses on the transformed data, and back transform at the end $\rightarrow$ back transform is sensitive

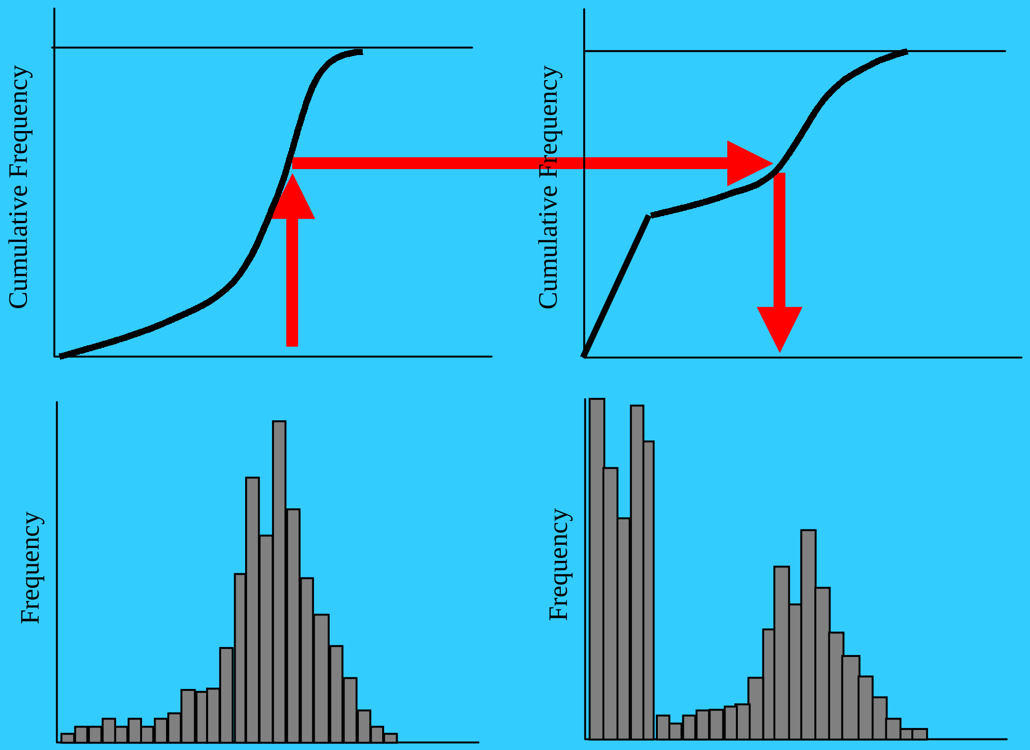
- Many geostatistical techniques require the data to be transformed to a Gaussian or normal distribution.

The Gaussian RF model is unique in statistics for its extreme analytical simplicity and for being the limit distribution of many analytical theorems globally known as "central limit theorems"

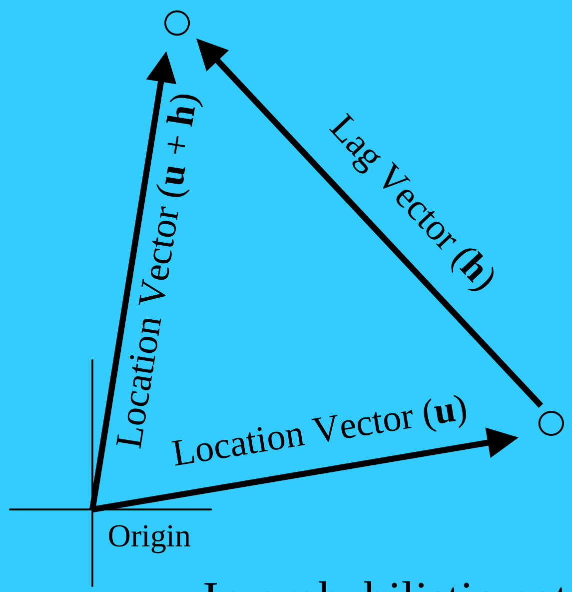
The transform to any distribution (and back) is easily accomplished by the quantile transform

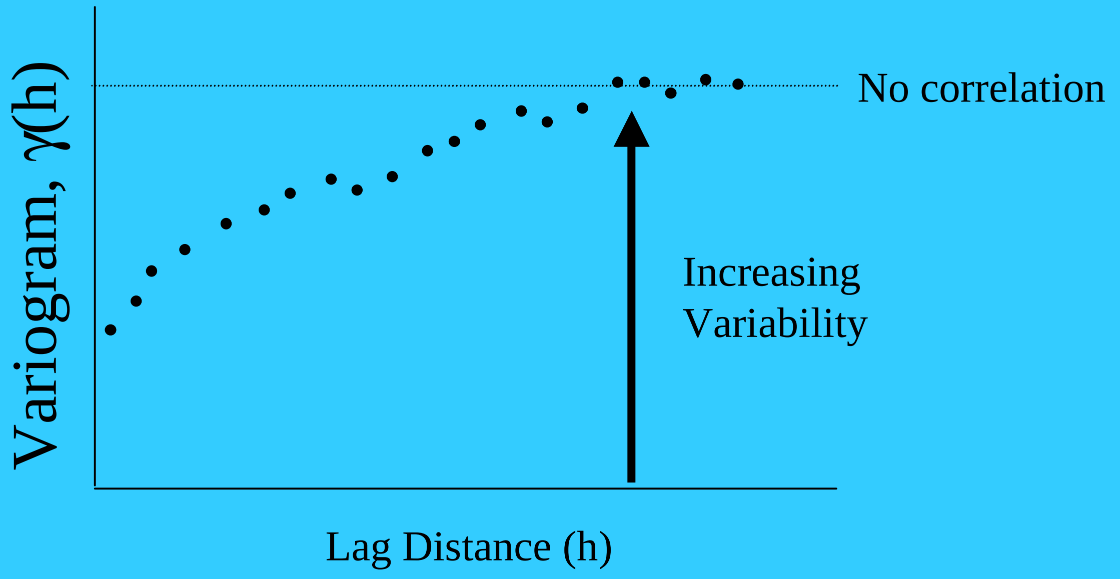
Normal Scores Transformation

- Many geostatistical techniques require the data to be transformed to a Gaussian or normal distribution:



Definition of the Variogram





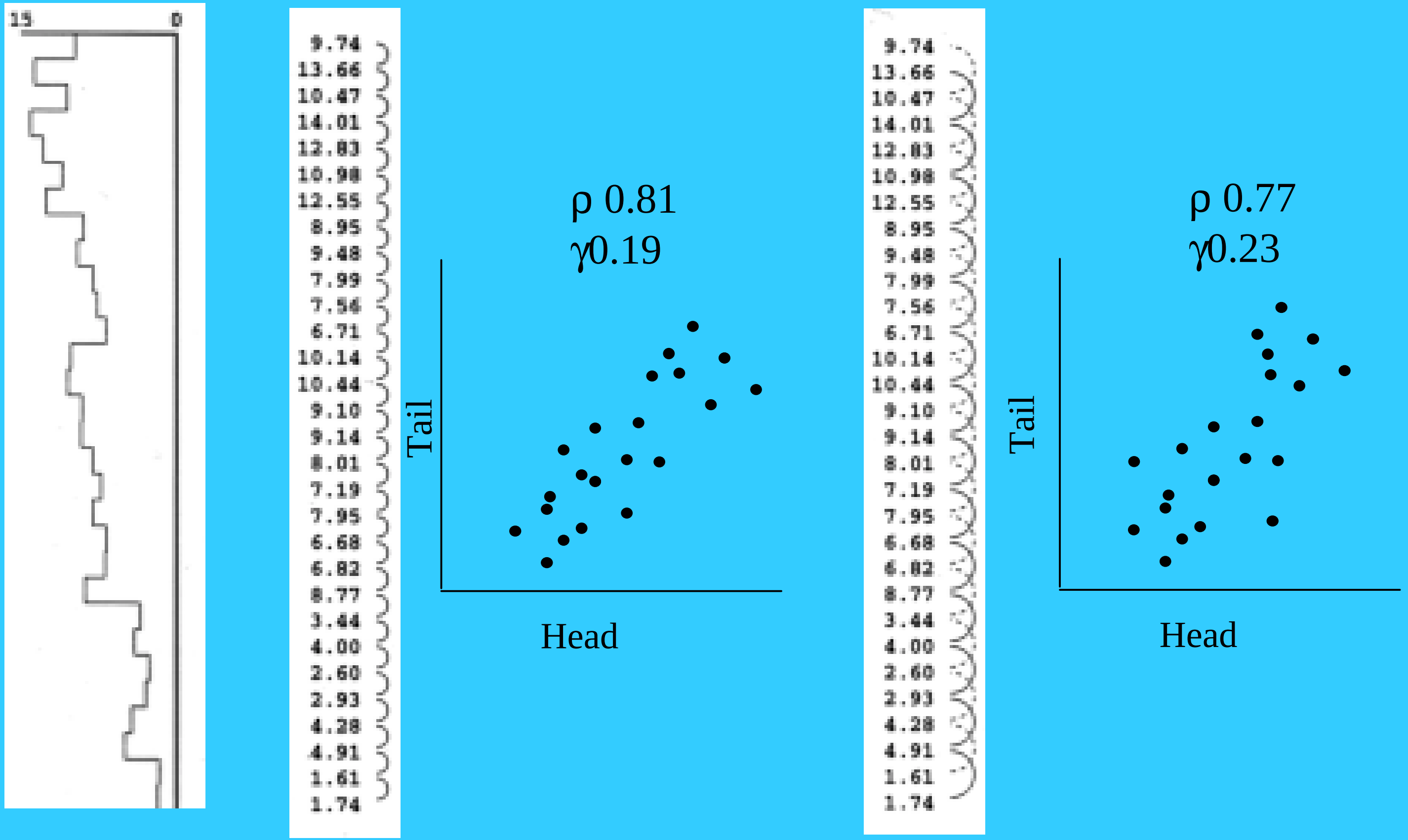
- In probabilistic notation, the variogram is defined as:
- $\mathbf{2\gamma(h) = E\{[Z(u) - Z(u+h)]^2\}}$ - for all possible locations **u**
- The variogram for lag distance **h** is defined as the average squared difference of values separated approximately by **h**:

$$\mathbf{2\gamma(h) = \frac{1}{N(h)} \sum_{N(h)} [z(u) - z(u+h)]^2}$$

where N(**h**) is the number of pairs for lag **h**

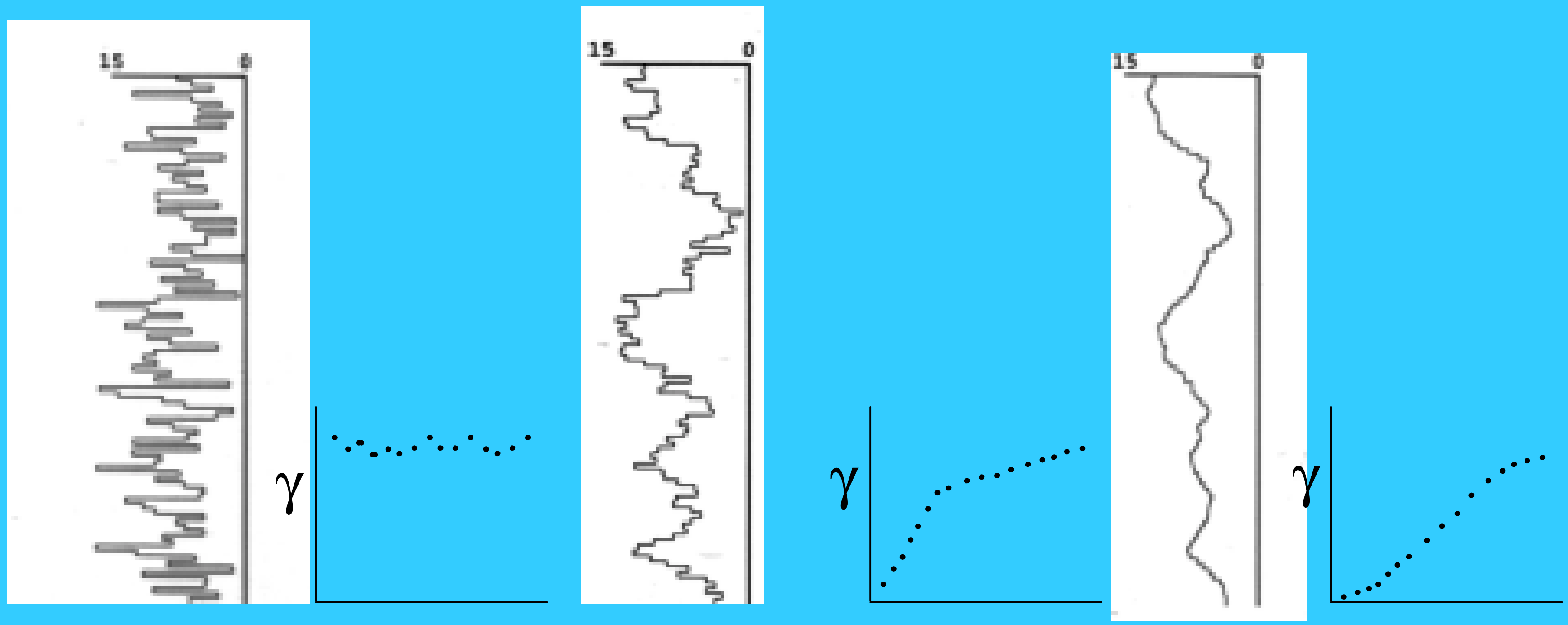
Variogram Calculation

- Consider data values separated by *lag* vectors



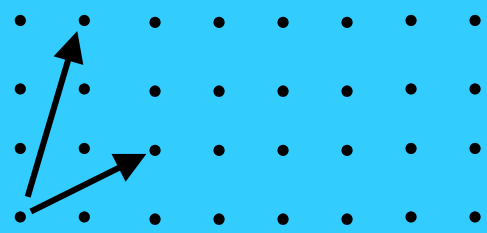
Spatial Description

The Variogram is a tool that <u>Quantifies Spatial Correlation</u>

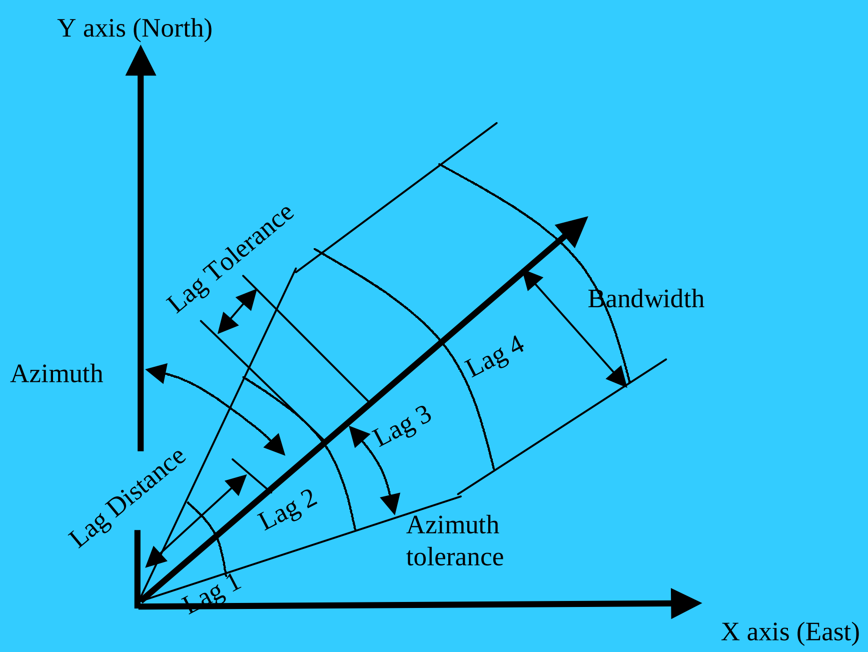


Calculating Experimental Variograms

- 2-D or 3-D, regular or irregular spaced
- Direction specification (regular):

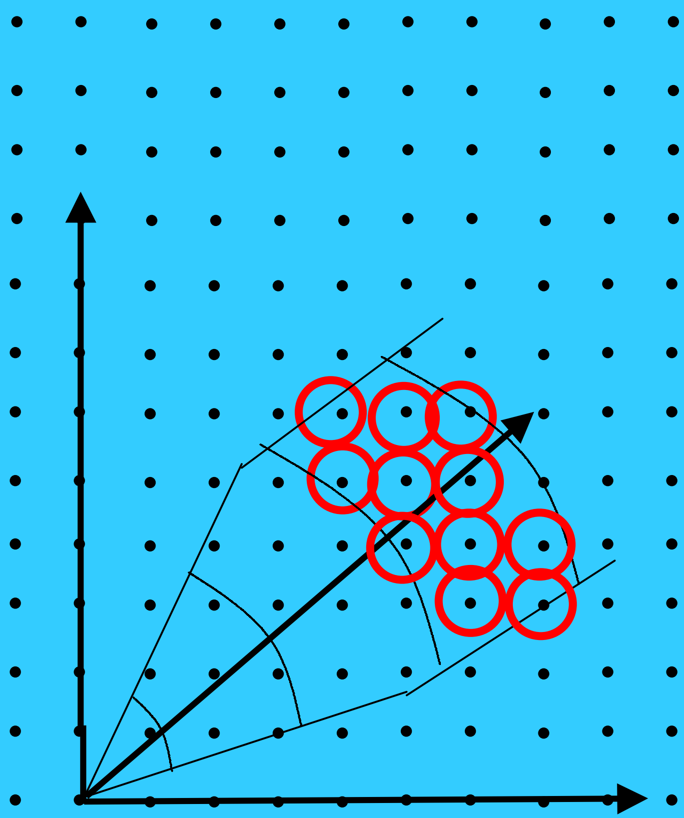


- Direction specification (irregular):



Calculating Experimental Variograms

Example: Starting With One Lag (i.e. #4)

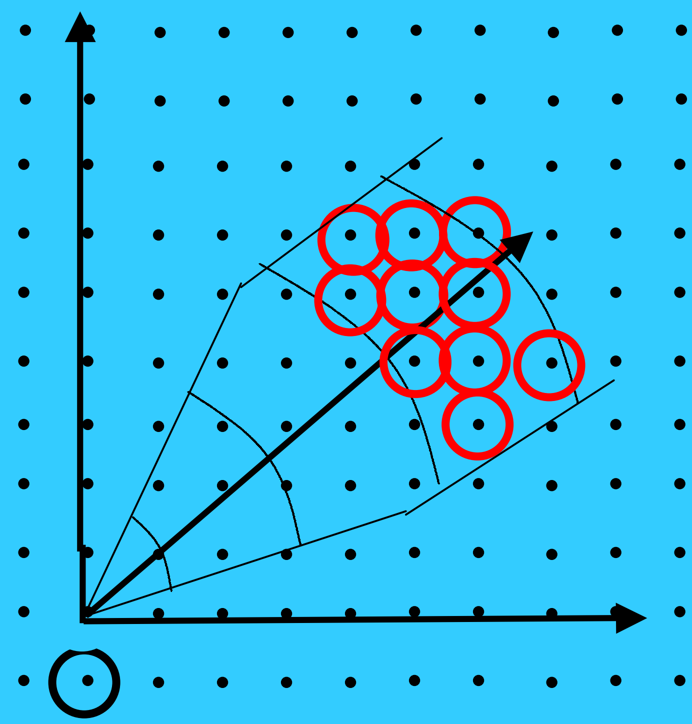


$$2\gamma(h) = \frac{1}{N(h)} \sum_{N(h)} [z(u) - z(\underline{u+h})]^2$$

Start at a node, and compare value to all nodes which fall in the lag and angle tolerance.

...

Calculating Experimental Variograms



$$2\gamma(\boldsymbol{h}) = \frac{1}{N(\boldsymbol{h})} \sum_{N(\boldsymbol{h})} [z(\boldsymbol{u}) - z(\boldsymbol{u}+\boldsymbol{h})]^2$$

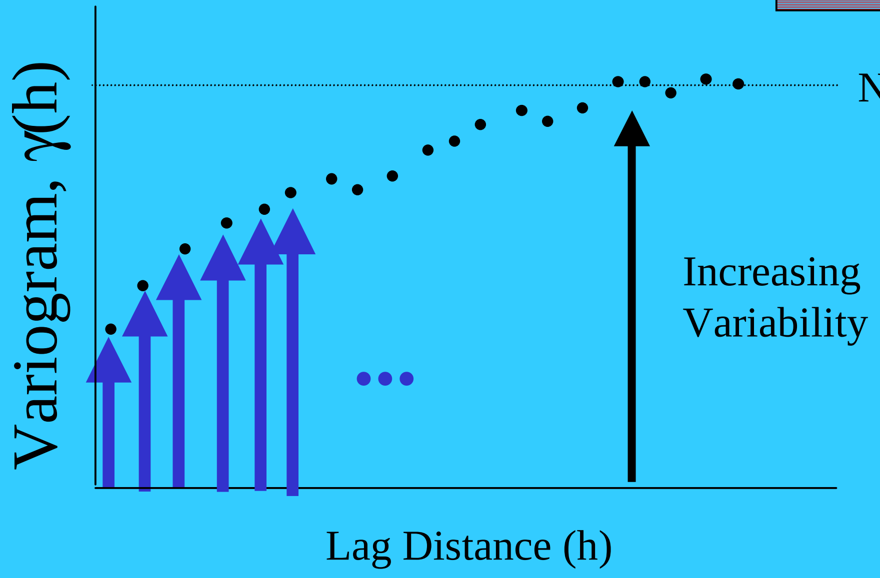
Move to next node.

…

Calculating Experimental Variograms

Now Repeat for All Nodes

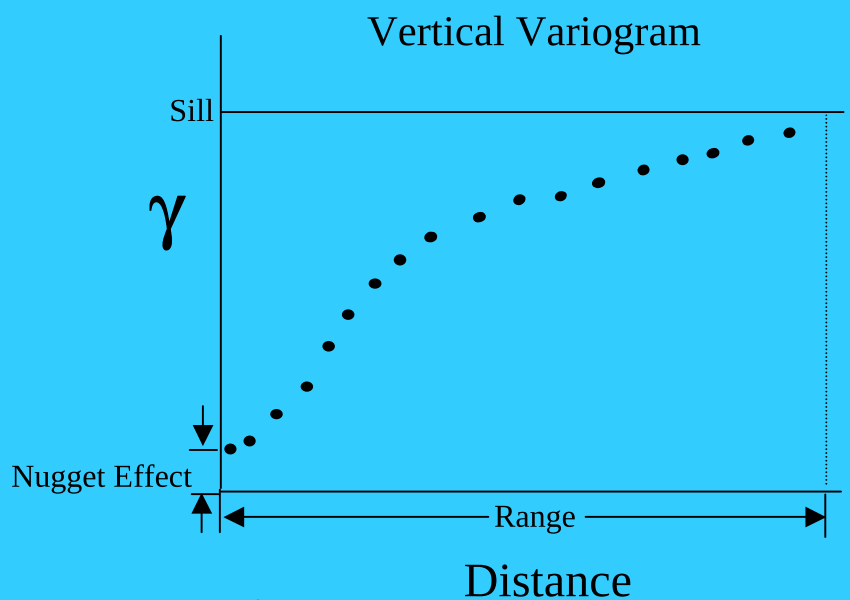
And Repeat for All Lags



Variogram Calculation Options

- Data variable (transformed?) and coordinates (transformed?)
- Number of directions and directions:
  - compute the vertical variograms in one run and the horizontal variograms in another
  - often choose three horizontal directions: *omnidirectional*, “major” direction, and perpendicular to major direction
  - azimuth angles are entered in degrees clockwise from north
- Number of lags and the lag separation distance:
  - lag separation distance should coincide with data spacing
  - the variogram is only valid for a distance one half of the field size $\mapsto$ choose the number of lags accordingly
- Number and type of variograms to compute:
  - there is a great deal of flexibility available, however, the traditional variogram applied to transformed data is adequate in 95% of the cases
  - typically consider one variogram at a time (each variogram is computed for all lags and all directions)

Interpreting Experimental Variograms

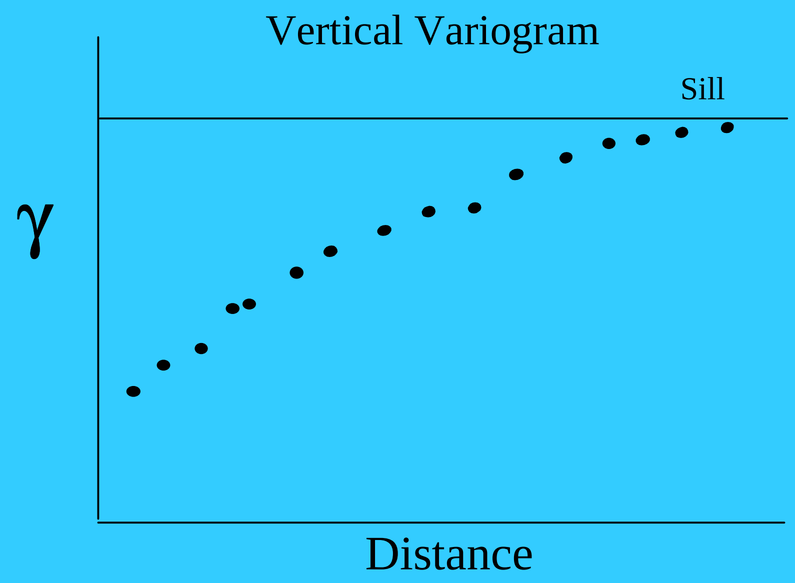


- sill = the variance (1.0 if the data are normal scores)
- range = the distance at which the variogram reaches the sill
- nugget effect = sum of geological microstructure and measurement error
  - Any error in the measurement value or the location assigned to the measurement translates to a higher nugget effect
  - Sparse data may also lead to a higher than expected nugget effect

Challenges in Variogram Calculation

- Short scale structure is most important
  - nugget due to measurement error should not be modeled
  - size of geological modeling cells
- Vertical direction is typically well informed
  - can have artifacts due to spacing of core data
  - handle vertical trends and areal variations
- Horizontal direction is not well informed
  - take from analog field or outcrop
  - typical horizontal vertical anisotropy ratios

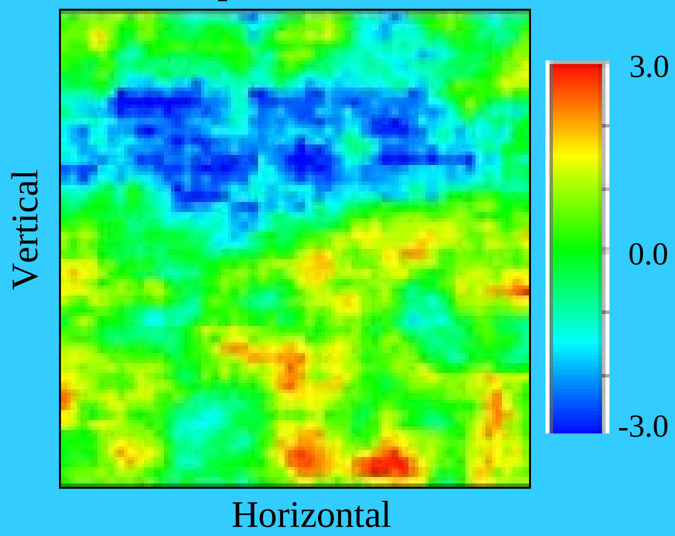
Interpreting Experimental Variograms



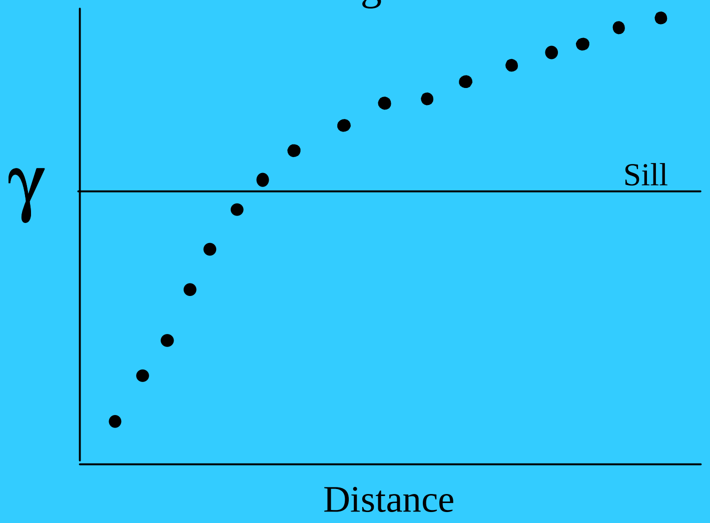
- vertical permeability variogram
- sill: clearly identified (variance of log *K* data)
- nugget: likely too high

Trend

Example Trend Data Set

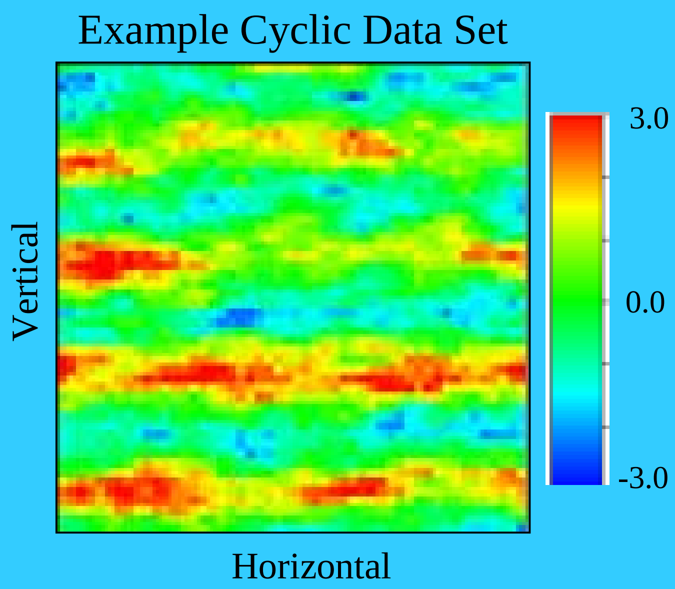


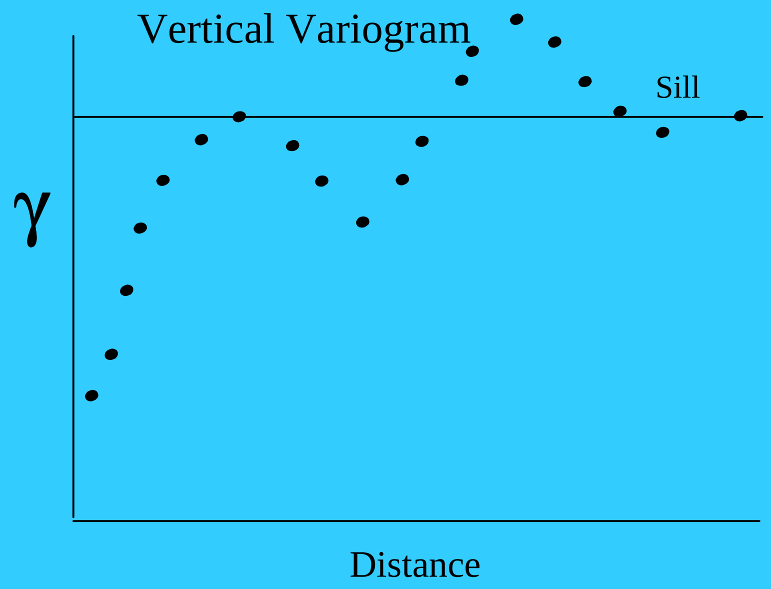
Vertical Variogram



- indicates a trend (fining upward, …)
- could be interpreted as a fractal
- model to the theoretical sill; the data will ensure that the trend appears in the final model
- may have to explicitly account for the trend in later simulation/modeling

Cyclicity

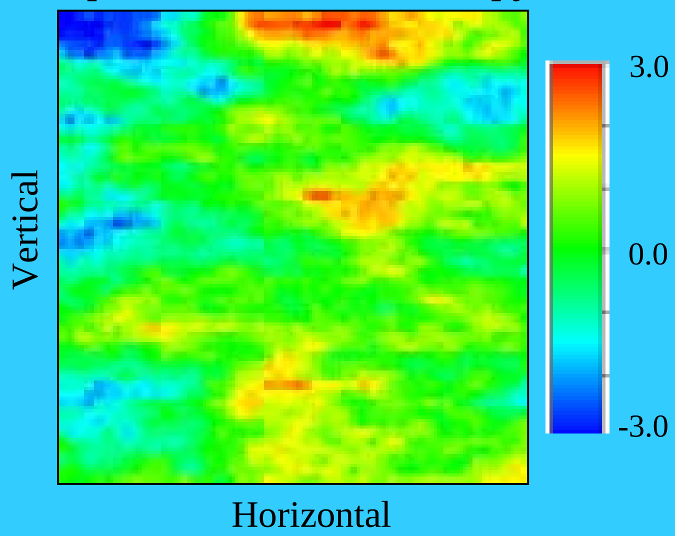


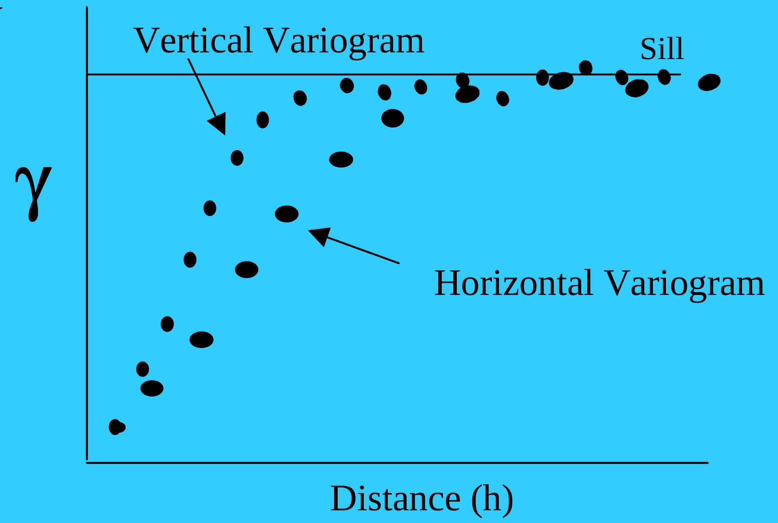


- cyclicity may be linked to underlying geological periodicity
- could be due to limited data
- focus on the nugget effect and a reasonable estimate of the range

Geometric Anisotropy

Example Geometric Anisotropy Data Set

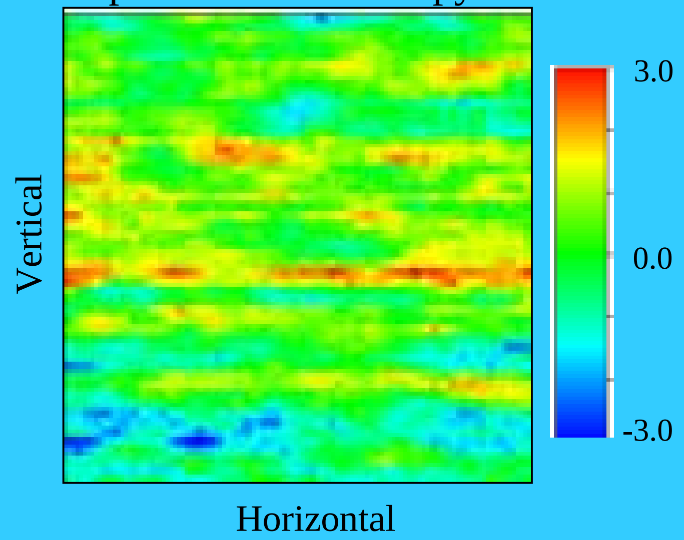


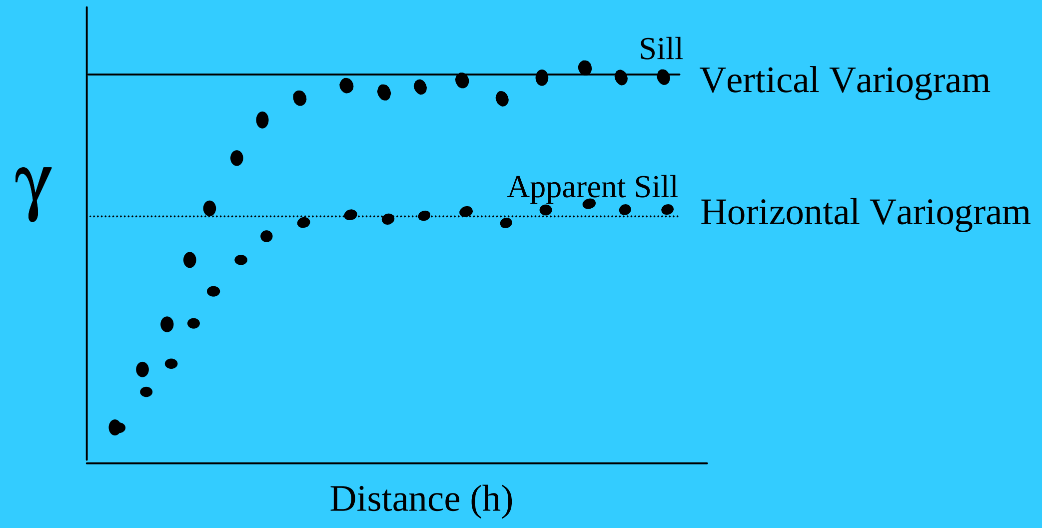


- Compare vertical sill with horizontal sill
- When the vertical variogram reaches a *higher* sill:
  - likely due to additional variance from stratification/layering
- When the vertical variogram reaches a *lower* sill:
  - likely due to a significant difference in the average value in each well $\mapsto$ horizontal variogram has additional between-well variance
- There are other explanations

Zonal Anisotropy

Example Zonal Anisotropy Data Set





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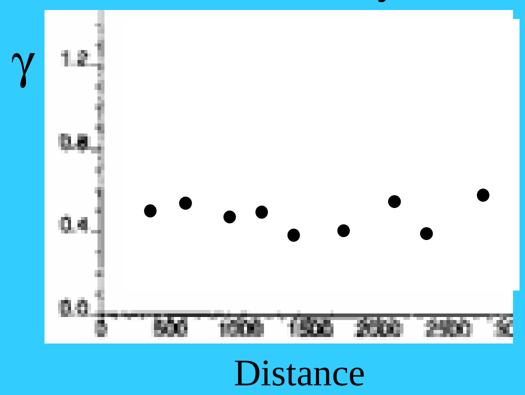
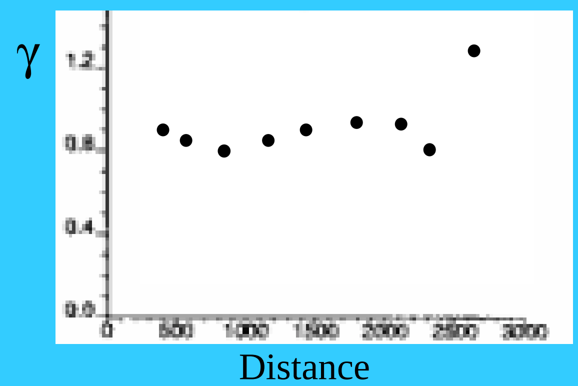
Horizontal Variograms

A few experimental horizontal variograms:

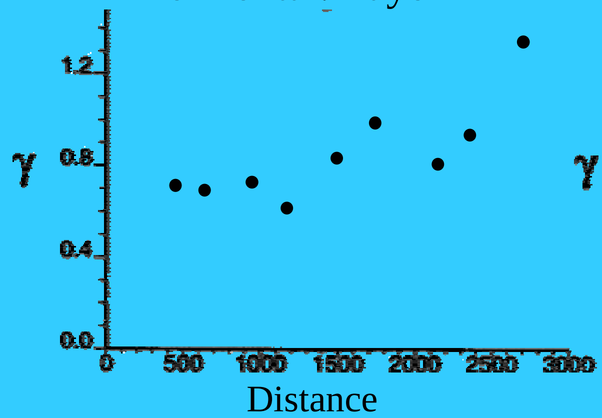
Horizontal: Layer 01



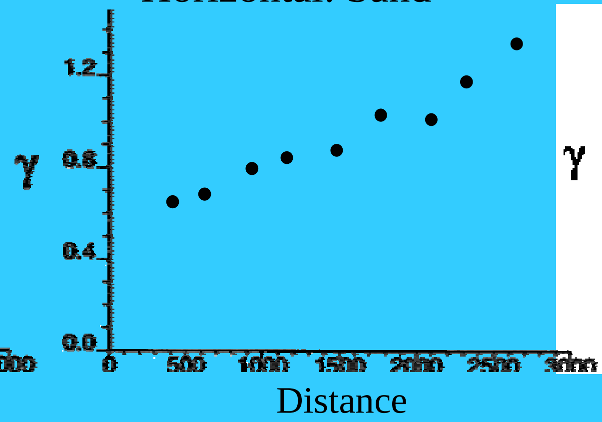
Horizontal: Layer 13



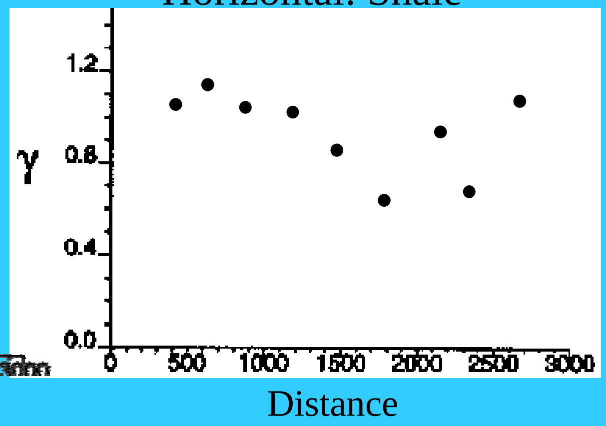
Horizontal: Layer 14



Horizontal: Sand



Horizontal: Shale



Noise is often due to scarcity of data in the horizontal direction.

Variogram Interpretation and Modeling

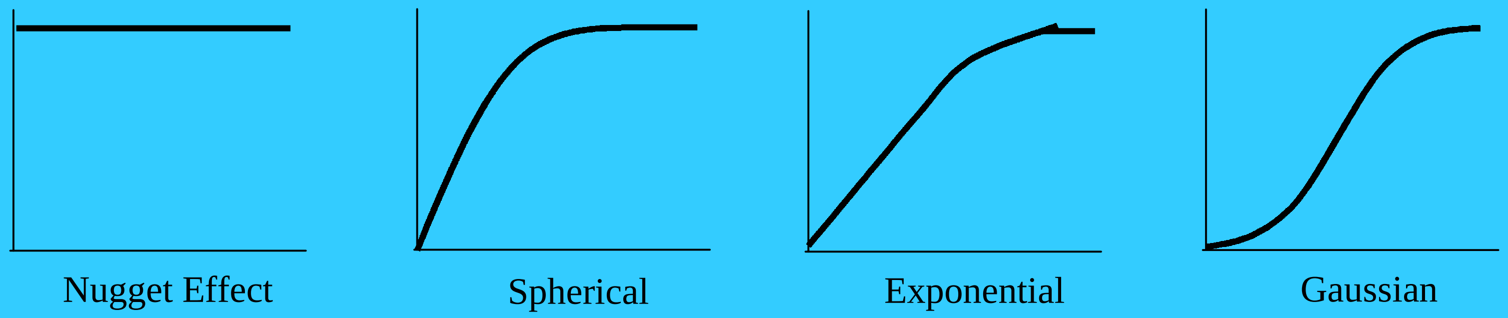
Key is to apply geologic knowledge to the experimental variogram and to build a legitimate (positive definite) variogram model for kriging and simulation (discussed later)

This ensures:

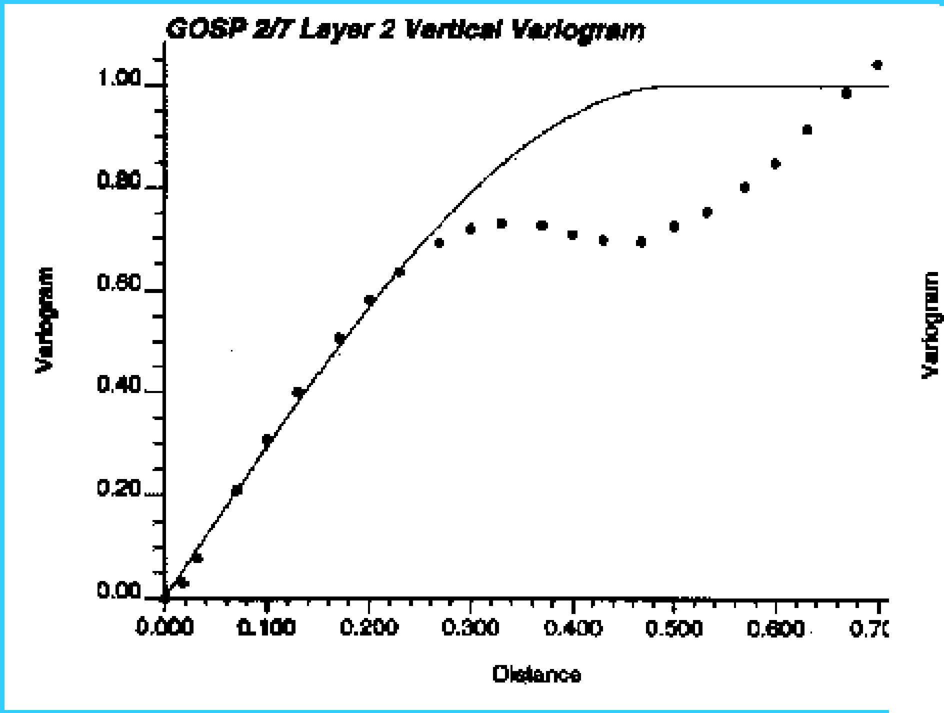
- that the covariance can be assessed over all lag vectors, **h**.
- that the variogram will be a legitimate measure of distance

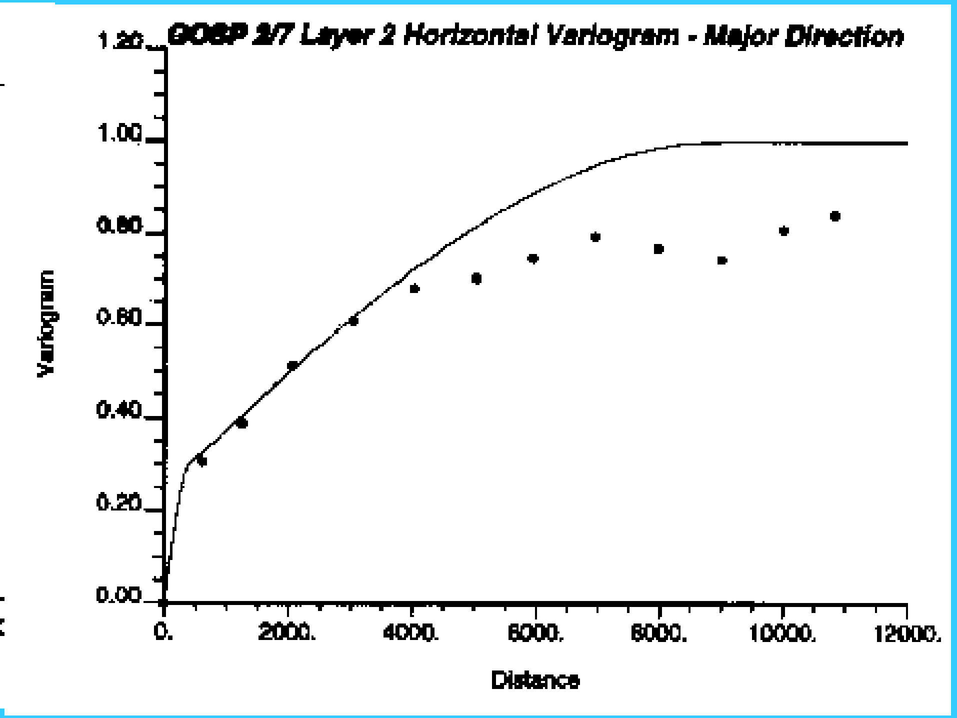
The sum of known positive definite models is positive definite. There is great flexibility in modeling variograms with linear combinations of established models.

Some common positive definite models:



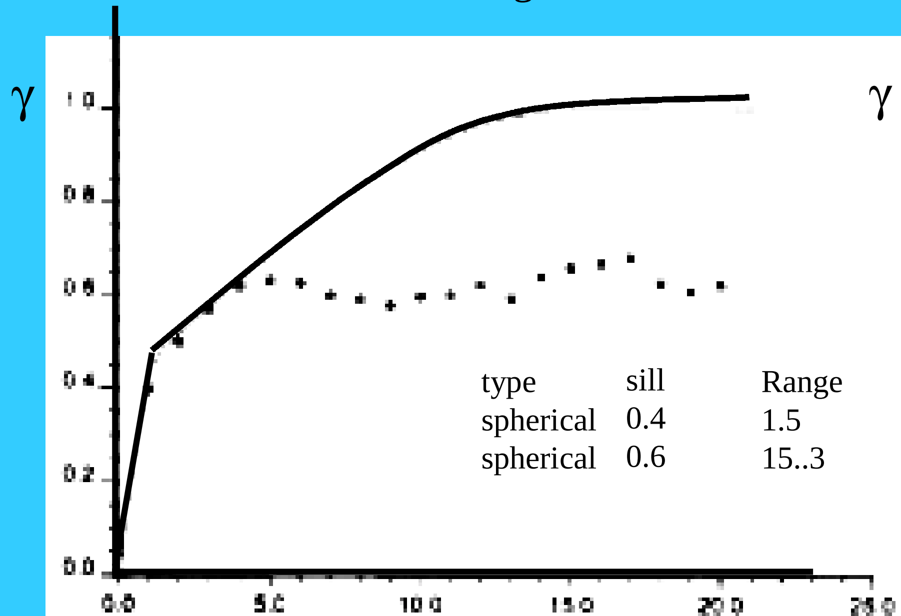
Horizontal Variograms





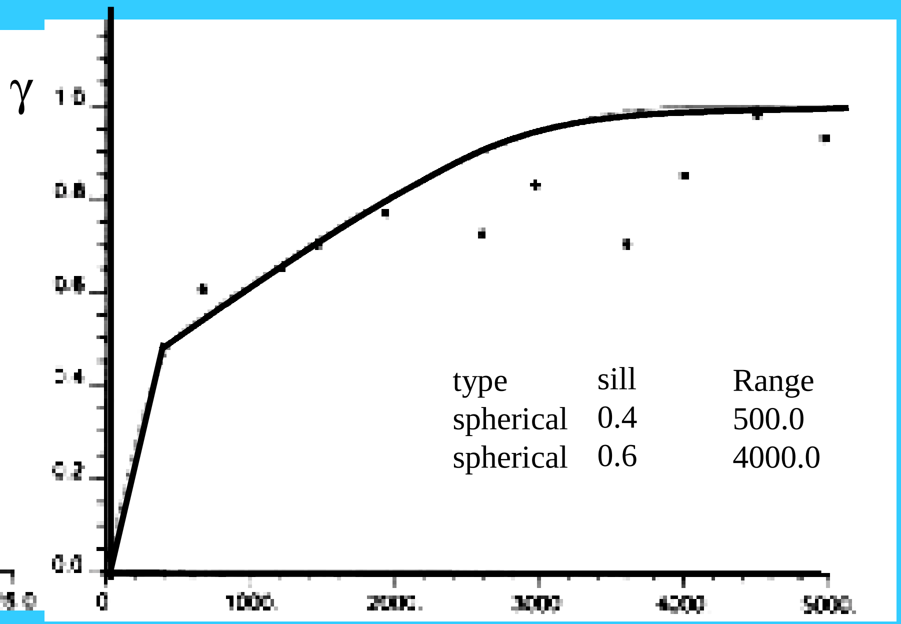
Porosity Variogram

Vertical Variogram



Distance

Horizontal Variogram



Distance

Summary

- Variogram is very important in a geostatistics study
- Measure of geological distance with respect to Euclidian distance
- Initial coordinate and data transformation
- Calculation principles
- Interpretation principles:
  - trend
  - cyclicity
  - geometric anisotropy
  - zonal anisotropy
- Variogram modeling is important (experimental points are not used)